1. (10 points): Suppose that \( f(x, y) = \frac{x^4 + 3y^3}{2x^3 - y^3} \). Does \( \lim_{(x,y)\to(0,0)} f(x, y) \) exist? If it exists, give its value. If it does not exist, briefly explain how you reached your conclusion.

2. (10 points): Find an equation of the form \( ax + by + cz = d \) for the plane tangent to the surface \( F(x, y, z) = y - x^2 + z^2 = 0 \) at the point \((4, 7, 3)\).

3. (20 points):
   (a) (12 points): Suppose that 
   \[ u = z \sin x + zy^3 - y^2, \quad x = e^t, \quad y = \ln(t+1), \quad z = 3t^5 + 2t + 1. \]
   Use the Chain rule to compute \( du/dt \). You do not need to put the answer in terms of \( t \) alone; you may use the variables \( x, y, \) and \( z \) as well.
   (b) (8 points): Suppose that 
   \[ z = F(x, y, t), \quad x = G(t), \quad y = H(s, t). \]
   Use partial derivatives of \( F, G, \) and \( H \) to write down the form of the chain rule that you would use to compute \( \partial z/\partial t \). (Use a tree diagram to show variable dependencies.)

4. (20 points): Let \( f(x, y, z) = xy + z^2 \).
   (a) (13 points): Find the directional derivative of \( f(x, y, z) \) at \((1, 1, 1)\) in the direction of \((5, -3, 3)\).
   (b) (7 points): Give a unit vector in the direction in which \( f \) decreases most rapidly at \((1, 1, 1)\).

5. (25 points): Let \( f(x, y) = x^2 - 6xy + y^2 + 4x + 4y \).
   (a) (12 points): Use the second derivative test to locate all local maxima, minima, and saddle points of \( f(x, y) \).
   (b) (13 points): Let \( D = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 5\} \). Find the absolute maximum and minimum of \( f(x, y) \) on \( D \). Justify your conclusions.

6. (15 points): Use Lagrange multipliers to find the point(s) \((x, y)\) which maximize and minimize \( f(x, y, z) = x + 2y + 8z \) subject to the constraint \( g(x, y, z) = x^2 + y^2 + 2z^2 = 37 \).