Math 241, Quiz 12. 11/14/11. Name: _______________________

- Read problems carefully. Show all work. No notes, calculator, or text.
- There are 15 points total.

1. §15.6, #9 (5 points): Evaluate \[ \iiint_{E} 2x \, dV, \] where 
   \[ E = \{(x, y, z) : 0 \leq y \leq 2, \ 0 \leq x \leq \sqrt{4-y^2}, \ 0 \leq z \leq y \}. \]

   **Solution:** We have
   \[
   \iiint_{E} 2x \, dV = 2 \int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^y x \, dz \, dx \, dy = 2 \int_0^2 \int_0^{\sqrt{4-y^2}} \left( \frac{x^2 y}{2} \right) \, dx \, dy
   \]
   \[
   = 2 \int_0^2 \int_0^{\sqrt{4-y^2}} xy \, dx \, dy = 2 \int_0^2 \left( \left. \frac{x^2 y}{2} \right|_0^{\sqrt{4-y^2}} \right) \, dy
   \]
   \[
   = \int_0^2 (4 - y^2)y \, dy = \int_0^2 (4y - y^3) \, dy = \left( \frac{4y^2}{2} - \frac{y^4}{4} \right) \big|_0^2 = 4.
   \]

2. §15.6, #21 (5 points): Set up a triple integral to compute the volume of the solid enclosed by the cylinder \[ x^2 + y^2 = 9 \] and the planes \[ y + z = 5 \] and \[ z = 1. \] Do not evaluate.

   **Solution:** We have
   \[
   \text{Volume}(E) = \iiint_E dV = \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_1^5 dz \, dy \, dx.
   \]

3. §15.6, #31 (5 points): Suppose that \( E \) is the solid bounded by the surfaces
   \[ y = x^2, \quad z = 0, \quad y + 2z = 4. \]

   **Set up** the integral \[ \iiint_E f(x, y, z) \, dV \] using \( dV = dx \, dy \, dz. \) Do not evaluate.

   (Note: You should view \( D \), the projection of \( E \), in the \( yz \)-plane.)

   **Solution:** We have
   \[
   \iiint_E f(x, y, z) \, dV = \iint_D \left( \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) \, dx \right) \, dA = \int_0^2 \int_0^{4-2z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) \, dx \, dy \, dz.
   \]