1. { 12 points } Use any method to determine whether the series converges or diverges.

(A) \[ \sum_{k=1}^{\infty} \frac{1}{\sqrt{(k - 1)k(k + 1)}} \]

\[ a_k = \frac{\sqrt{k^3}}{\sqrt{(k - 1)k(k + 1)}} \rightarrow 1 \]

and therefore the series is convergent together with the \( p \)-series for \( p = \frac{3}{2} > 1 \).

(B) \[ \sum_{k=1}^{\infty} \left( \frac{1}{\ln k} \right)^k \]

\[ a_k^\frac{1}{k} = \left( \frac{1}{\ln k} \right)^\frac{k}{k} = \frac{1}{\ln k} \rightarrow 0 < 1 \]

and therefore the series is convergent by the root test.

(C) \[ \sum_{k=1}^{\infty} \frac{(-1)^k k^3}{k^2 + 1} \]

\[ \lim_{k \to \infty} \frac{(-1)^k k^3}{k^2 + 1} \neq 0 \]

and therefore the series is divergent.

(D) \[ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{\frac{3}{2}}} \]

\( \{ a_k \} = \left\{ \frac{1}{k^{\frac{3}{2}}} \right\} \) is a decreasing sequence and \( \lim_{k \to \infty} \frac{1}{k^{\frac{3}{2}}} = 0 \). Thus, the alternating series converges.