1. { 1 point for each correct answer } For each of the following statements determine whether it is true or false. It is not necessary to justify your answer.

(A) The difference of any two odd integers is even.  \textbf{true}
Let \( n = 2p + 1 \) and \( m = 2q + 1 \) be two arbitrary odd integers. Then \( n - m = 2p - 2q = 2(p - q) \) is even.

(B) For all integers \( m \), if \( m > 2 \) then \( m^2 - 4 \) is composite.  \textbf{false}
Counterexample: for \( m = 3 \) we have \( m^2 - 4 = 5 \) which is a prime number.

(C) For all nonnegative real numbers \( a \) and \( b \), \( p^a + b = p^a + b^2 \).
\textbf{false}
Counterexample: for \( a = 1 \) and \( b = 1 \) we have \( \sqrt{a + b} = \sqrt{2} \neq \sqrt{a} + \sqrt{b} = 2 \).

(D) The product of any two even integers is a multiple of 4. \textbf{true}
Let \( n = 2p \) and \( m = 2q \) be two arbitrary even integers. Then \( n \cdot m = 2p \cdot 2q = 4pq \) is a multiple of 4.

(E) For all integers \( n \), \( n(6n + 3) \) is divisible by 3. \textbf{true}
We have \( n(6n + 3) = 3(2n^2 + 3n) \) which by the definition is divisible by 3.

(F) For all integers \( a \) and \( b \), if \( a \mid 10 \) then \( a \mid 10 \) or \( a \mid b \). \textbf{false}
Counterexample: for \( a = 4 \) and \( b = 2 \) we have \( a = 4 \mid 20 = 10b \) but \( a = 4 \nmid 10 \) and \( a = 4 \nmid 2 = b \).

2. { 4 points } Prove the statement.

The square of any odd integer is odd.

Let \( n = 2p + 1 \) be an arbitrary odd integer. Then
\[ n^2 = 4p^2 + 4p + 1 = 2(2p^2 + 2p) + 1 \]
is odd by the definition.

Bonus Problem. { 5 points } Prove the statement.

Given any two rational numbers \( r \) and \( s \) with \( r < s \), there is a rational number \( x \) such that \( r < x < s \).
In order to prove the statement, we have to find a rational number \( x \) which satisfies \( r < x < s \). For the number \( x = \frac{r+s}{2} \) we have
\[ r = \frac{r+r}{2} < \frac{r+s}{2} < \frac{s+s}{2} = s. \]
It remains to prove that \( x \) is rational. Let \( r = \frac{a}{b} \) and \( s = \frac{c}{d} \) where \( a, b \neq 0, c, \) and \( d \neq 0 \) are integers. Then
\[ x = \frac{r+s}{2} = r = \frac{a}{2b} + s = \frac{c}{2d} = \frac{ad+cb}{2bd}, \]
where \( ad+cb \) and \( 2bd \neq 0 \) are integers. Therefore \( x \) is rational.