

To study for the final exam of Math 141, your most valuable tools are the following.

- past exams / past practice exams
- this packet

1: Answer the following True or False questions.

- _____ (a) A function is continuous at $x = c$ if the limit as $x \rightarrow c$ exists *and* $f(c)$ exists.
- _____ (b) The inverse of $y = \tan(x)$ is $y = \frac{1}{\tan(x)}$.
- _____ (c) All functions have inverses.
- _____ (d) A function that is continuous on the interval (a, b) is *always* differentiable on (a, b) .
- _____ (e) A critical point in a function's domain only occurs when the derivative is zero.
- _____ (f) All functions are Riemann integrable.
- _____ (g) $\int_0^5 x^2 dx$ gives the area under the curve x^2 and above the x -axis on the interval $(0, 5)$.
- _____ (h) A function that is differentiable on an interval (a, b) is *always* continuous on (a, b) .
- _____ (i) For a continuous function $f(x)$, a point $x = a$ cannot be both a critical point and a point of inflection.
- _____ (j) If $f(b) = f(a)$ then f must be continuous on $[a, b]$.
- _____ (k) If $f'(x) = g'(x)$, then $f(x) = g(x)$.

2: Using any method that we have learned in this class, compute the following limits.

(a) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$

(b) $\lim_{x \rightarrow -\infty} \frac{2x^2 + 3}{5x^2 + 7}$

(c) $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

(d) $\lim_{x \rightarrow \infty} \frac{x + 1}{x^4 + x - 1}$

(e) $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$

(f) $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$

3: Compute the derivatives of the following.

(a) $f(x) = \frac{x^2 + x + 1}{x + 3}$

(b) $g(x) = (x + 9)(\sin(x) + \cos(x))$

(c) $h(x) = 2e^{x^2+x}$

(d) $b(x) = \ln(2^x + \cos(2x))$

4: Find the slope of the folium of Descartes $x^3 + y^3 - 9xy = 0$ at the points $(4, 2)$ and $(2, 4)$.

5: A hot air balloon (full of calculus books) is rising vertically above a level, straight road at a constant rate of 1ft/sec. Just when the balloon is 50ft above the ground, Alicia, moving at a constant rate of 5ft/sec passes under it. How fast is the distance $s(t)$ between the balloon and Alicia increasing 3 seconds later?

6: Consider the function $f(x) = x^4 - 2x^3 + 1$. Determine all critical points of $f(x)$, and write the intervals where $f(x)$ is increasing or decreasing. Further, find all points of inflection of $f(x)$ and determine on what intervals $f(x)$ is concave up or concave down.

5: Compute the following integrals.

(a) $\int 2(2x + 4)^{10} dx$

(b) $\int \frac{(1 + \sqrt{x})^{1/3}}{\sqrt{x}} dx$

(c) $\int \sin(3x + 1) dx$

(d) $\int_3^5 \frac{1}{x \ln(x)} dx$

(e) $\int_0^\pi 1 + \cos(x) dx$

(f) $\int_1^2 x(x - 3) dx$