4.3: Global Maxima and Minima

2.5: Marginal Cost and Revenue

4.4: Profit, Cost, and Revenue

Maximizing Profit

Maximizing Revenue

Math 122

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Calculus for Business Administration and Social Sciences
4.3: Global Maxima and Minima

Maximizing Profit
Maximizing Revenue
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OUTLINE

1. 4.3: GLOBAL MAXIMA AND MINIMA

2. 2.5: MARGINAL COST AND REVENUE

3. 4.4: PROFIT, COST, AND REVENUE
   - Maximizing Profit
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**DEFINITION 1**

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Example

Find the global extrema of \( f(x) = x^3 - 9x^2 - 48x + 52 \) on \([-5, 14]\).
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\Rightarrow f'(x) = 3(x + 2)(x - 8)
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Maximum: \((14, 360)\).

Minimum: \((-2, 104)\).
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**Remark 1**

Critical points occur whenever marginal cost equal marginal revenue, or one of marginal cost/revenue is undefined.
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Find the quantity which maximizes profit for the given revenue and cost functions on \([0, 1000]\)

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R(q) = 5q - 0.003q^2 \\
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\[ R(65) = 25,350. \]